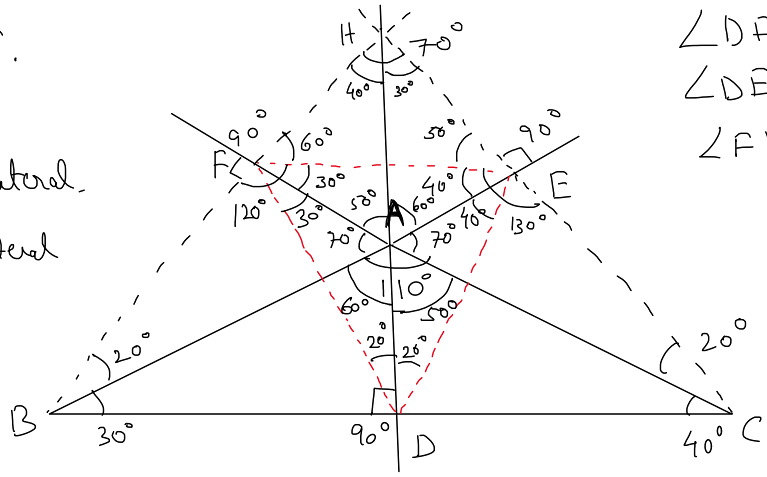
Geometry 15

10 August 2024 17:35

Q) ΔABC is a obtuse triangle. Angles are $110°$, $30°$, $40°$. Let H be the orthocentre.

Find the angles of $\Delta D\hat{E}F$.



$\angle DFE = 60°$

$\angle DEF = 80°$

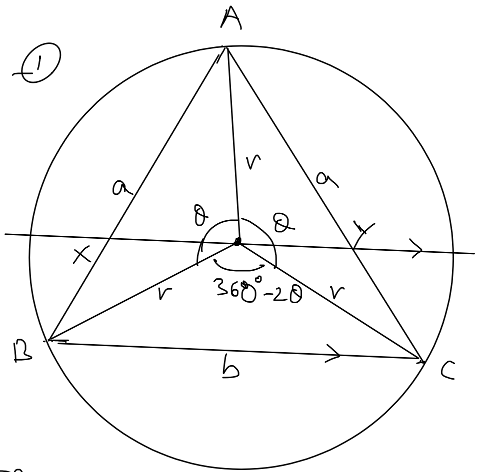
$\angle FDE = 40°$

Ans:- HFAE is a cyclic quadrilateral.

FECB is cyclic quadrilateral

$\Rightarrow \angle EFB = 120°$

Q) The length of two sides of a tringle are equal to a and the third side length is b. Find the length of radius of circumcircle of this triangle.



not needed

$\cancel{\boxed{b = 2a}}$

$\boxed{b < 2a}$

Ans:- $r^2 + r^2 - 2r^2\cos\theta = a^2 = 2r^2(1-\cos\theta)$ — ①

$r^2 + r^2 - 2r^2\cos(360° - 2\theta) =$

$2r^2(1 - \cos 2\theta) = b^2$ — ②

$\dfrac{②}{①} \Rightarrow \dfrac{2(1-\cos^2\theta)}{1-\cos\theta} = \dfrac{b^2}{a^2}$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$1 - \cos 2\theta = 1 - \cos^2\theta + \sin^2\theta = 2(1-\cos^2\theta)$

$\Rightarrow \dfrac{2\cancel{(1-\cos\theta)}(1+\cos\theta)}{\cancel{1-\cos\theta}} = \dfrac{b^2}{a^2}$

$\Rightarrow 2(1+\cos\theta) = \dfrac{b^2}{a^2} \Rightarrow \cos\theta = \dfrac{b^2}{2a^2} - 1 \xrightarrow[\text{value}]{\text{max}} < \dfrac{(2a)^2}{2a^2} - 1 = 1$

Putting in ① the value of $\cos\theta \rightarrow 2r^2\left(1 - \dfrac{b^2}{2a^2} + 1\right) = a^2$

$\Rightarrow r^2 = \dfrac{a^2 a^2}{4a^2 - b^2}$

$\Rightarrow r = \dfrac{a^2}{\sqrt{4a^2 - b^2}} \longrightarrow b < 2a \Rightarrow$

$b^2 < 4a^2$

$\Rightarrow r$ is real valued

Home Work

Q) A straight line passing through the point A of a square ABCD intersects side CD at E and line BC at F.

Prove that $\frac{1}{AE^2} + \frac{1}{AF^2} = \frac{1}{AB^2}$